An overview of oscillator jitter

1. Introduction

A basic measure of an oscillator's performance is its frequency stability (or instability depending on one's point of view). In the long term (timescales of days or years), we refer to frequency changes as aging. On moderate timescales (seconds), the stability of an oscillator is often characterized in terms of its Allan variance [1]. On short timescales (usually less than one second), frequency stability is often characterized in terms the fluctuations of the phase of the signal (phase noise) or the fluctuations in timings of its transitions or the periods of its cycles (jitter).

In this note, we provide an overview of timing, period, and cycle-to-cycle jitter in oscillators. Our discussion is brief, but should be enough to introduce most readers to the basic ideas in the subject. For further discussion of these topics, we refer the reader to references [2-4]. Further, for an overview of Statek's measurement method, we refer the reader to reference [5].

In Sec. 2, we discuss the basic concepts of timing jitter, period jitter, and cycle-to-cycle jitter. In Sec. 3, we revisit these concepts taking into account some measurement issues. In Sec. 4, we discuss the analysis of jitter, in particular its measures, random and non-random jitter, its sources, and the units used to express jitter. In Sec. 5, we discuss jitter in quartz crystal oscillators. In Sec. 6, we discuss system induced jitter. Lastly, in Sec. 7, we derive some simple relationships between the various jitter types that hold in the case that the jitter timings are uncorrelated with one another.

2. Jitter: Basic concepts

In this section, we present the basic concepts without regard to how measurements are made. While somewhat idealistic, this provides a clarity that would otherwise be blurred. In Sec. 3, we revisit these ideas once again, this time making some consideration for practical measurements.

Consider a nearly periodic signal whose frequency varies about some nominal value υ_0. Roughly speaking, jitter refers to the fast changes in the frequency of this signal. Slow frequency variations, such as that due to temperature changes or even aging are referred to as wander and are not of interest—they are not jitter. Although definitions vary depending on the application, frequency variations slower than 10 Hz are considered wander and those faster than this are considered jitter. So, let us suppose that our signal has no wander. (In Sec. 3, we discuss how we relax this requirement in practice.)

We are interested in the sequence of the times at which our signal makes transitions. For example, if this is a square-wave oscillator, with the voltage swinging between ground and the supply voltage V_{DD}, we can consider the times at which the output voltage rises through the mid-point voltage $\frac{1}{2}V_{\mathrm{DD}}$. Likewise, if this is an ac-coupled sine-wave output (and hence has zero mean), we can consider the times at which the voltage rises through the ground voltage.

So, for such a signal, we construct the increasing sequence of times t_k for $k = 0 \ldots N$ representing the times at which our signal makes a crossing. Viewing our signal as a clock, we can think of these as being the times at which our clock "ticks". Were our signal exactly periodic with period $\tau = 1/\upsilon_0$, we would have $t_k = \mathcal{T}_k$ where

$$\mathcal{T}_k = t_0 + k\tau \,. \tag{1}$$

However, because of jitter this won't be exactly the case.

2.1. Timing jitter, TIE, phase jitter

In some systems, it is important to keep two signals synchronized so that they are phase locked. As discussed in Sec. 3, this is normally done using a phase-locked loop (PLL) so that one oscillator follows the other. However, while a PLL can adjust for the wander of the oscillators, it cannot adjust for fast frequency changes, i.e., jitter. In this case, we want a measure of how much the two signals may become out-of-step or out-of-phase.

Timing jitter measures the lateness or earliness of the transitions (ticks) of our signal at any given time in its history. That is, timing jitter is the difference $\mathcal{J}_k$ in time between when a transition occurs t_k and the time it should have occurred $\mathcal{T}_k$, i.e.,

$$\mathcal{J}_k = t_k - \mathcal{T}_k \,, \tag{2}$$

where $\mathcal{T}_k$ is defined by equation (1).

Timing jitter is commonly referred to as time-interval error (TIE) as this is just the error in the transition times of our signal. Further, at least for nearly sinusoidal signals, the root-mean-square timing jitter $\mathcal{J}_{\text{rms}}$ is related to the root-mean-square phase fluctuations (phase noise) φ_{rms} as follows.

$$\mathcal{J}_{\text{rms}} = \tfrac{1}{2\pi}\varphi_{\text{rms}}\,\tau\,, \tag{3}$$

where

$$\varphi_{\text{rms}} = \sqrt{\int S_\varphi(f)\,\mathrm{d}f} \tag{4}$$

and $S_\varphi(f)$ is the spectral density of phase fluctuations (which is related to the conventional single-side band phase-noise spectrum $\mathcal{L}(f)$ by $\mathcal{L}(f) = \tfrac{1}{2}S(f)$) [1].

Because of this, timing jitter is often referred to as phase jitter (often limited to a frequency band of phase fluctuations), and jitter (of all types) is often expressed as a phase angle ($\varphi_k = 2\pi\,\mathcal{J}_k/\tau$), or as its fraction of a full period ($\mathcal{J}_k/\tau$).

2.2. *Period jitter*

Instead of looking at the effects of frequency variations in the long-term, we can look at its effects in the short-term. In particular, we look at the periods P_k of the individual cycles and study how they are distributed. That is, we study the sequence

$$P_k = t_{k+1} - t_k\,, \tag{5}$$

for $k = 0 \ldots N-1$.

Period jitter refers to the distribution of P_k. Were the signal perfectly periodic with period τ, then we would have $P_k = \tau$ for all k. However, we are more interested in the difference of these periods from the nominal period τ. So, instead we consider

$$j_k = P_k - \tau\,. \tag{6}$$

We refer to j_k as the period jitter of the k^{th} cycle. Notice that j_k has zero mean (in the long term). Further, note that

$$j_k = \mathcal{J}_{k+1} - \mathcal{J}_k\,. \tag{7}$$

That is, period jitter is the first-difference of the timing jitter.

2.3. *Cycle-to-cycle jitter*

Next, we consider how periods change from once cycle to the next. That is, we consider the sequence c_k given by

$$c_k = P_{k+1} - P_k\,. \tag{8}$$

for $k = 0 \ldots N-2$. Notice that

$$c_k = j_{k+1} - j_k\,, \tag{9}$$

i.e., cycle-to-cycle jitter is the first-difference of the period jitter (and is the second-difference of the timing jitter). See Figure 1.

This type of jitter measures very fast changes in the period. In fact, it measures changes on timescales of the period itself. For example, if the period were to vary slowly (compared to itself), we would have a wide spread in the period jitter j_k, but a much narrower spread in the cycle-to-cycle jitter c_k.

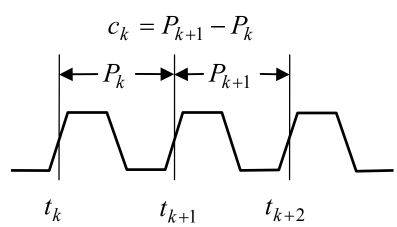

Figure 1—Three consecutive rising edges bounding two consecutive cycles. The cycle-to-cycle jitter is the change in the period from one cycle to the next.

2.4. *Summary*

Before moving on, it is worth summarizing the relationships between the various quantities discussed in this section. Define Δ to be the first difference operator, so that for a sequence q_k, Δq_k is the sequence given by

$$\Delta q_k = q_{k+1} - q_k\,. \tag{10}$$

Then, as noted above, we have the following simple relationships

$$\begin{array}{ccccc} t_k & \xrightarrow{\Delta} & P_k & \xrightarrow{\Delta} & c_k \\ \downarrow & & \downarrow & & \\ \mathcal{J}_k & \xrightarrow{\Delta} & j_k & \xrightarrow{\Delta} & c_k \end{array}\,. \tag{11}$$

That is $P_k = \Delta t_k$, $c_k = \Delta P_k$, $j_k = \Delta\mathcal{J}_k$, and $c_k = \Delta j_k$. Further, we arrive at $\mathcal{J}_k$ by subtracting off a linear expression from t_k, since we are interested in its variations about this exact linearity. Likewise, we arrive at j_k by subtracting off the nominal period from P_k, since we are interested in its variations about this nominal value. Lastly, as c_k naturally varies about zero, we did not need a modified version of it to study its variations.

3. Jitter: Measurement issues

In Sec. 2, we presented the basic concepts defining various types of jitter. While the ideas are rather straightforward, making actual measurements are rather involved.

In our definition of timing jitter, we compared our signal of interest to its idealized counterpart: an oscillator whose period was perfectly constant. Further, we did not worry about our oscillator wandering away in frequency (or phase) from its ideal. How do we accomplish this in practice?

One method for accomplishing these tasks is to use a reference signal. Ideally the reference signal is perfect (its frequency is constant and hence has no jitter), but in practice this is not the case. So, in the end we actually measure the combined jitter of both signals. Because of this, the jitter of the reference signal must be sufficiently low that its contribution is either ignorable or acceptable.

So, suppose we have such a reference signal with the same nominal frequency as our signal of interest. In practice, even if both oscillators have the same nominal frequency to very tight tolerances, the two won't be exactly the same. To eliminate any remaining difference we require that one of the two oscillators have some degree of frequency control (i.e., it is a VCXO). We then phase lock the controllable oscillator to the other with a long-time loop constant. This keeps their long-term frequencies the same (eliminating wander) while still allowing for short-term frequency variations.

Actually, what we've described allows both oscillators to wander in frequency in tandem. To keep both fixed, we would use a high-stability reference oscillator (e.g., an OCXO, perhaps locked to an external reference) and we would phase-lock the oscillator under study to this reference.

We are again interested in the sequence t_k of times at which our signal makes transitions. And likewise, we are interested in the sequence of times T_k at which our reference oscillators makes transitions.

For timing jitter, we now compare our signal to the reference signal. So, our measured timing jitter J_k is the difference in time between when a transition occurs and the time it should have occurred as determined by the reference signal, i.e.,

$$J_k = t_k - T_k \,. \tag{12}$$

So, our measured timing jitter J_k is related to the ideal timing jitter $\mathcal{J}_k$ by

$$\begin{aligned} J_k &= t_k - T_k \\ &= \left(t_k - \mathcal{T}_k\right) - \left(T_k - \mathcal{T}_k\right). \\ &= \mathcal{J}_k - \left(T_k - \mathcal{T}_k\right) \end{aligned} \tag{13}$$

Or, dropping the subscripts, we have

$$J_{\text{signal}} = \mathcal{J}_{\text{signal}} - \mathcal{J}_{\text{reference}} \,. \tag{14}$$

That is, the measured timing jitter is different from the ideal timing jitter by the amount of jitter in our reference signal. As a direct consequence of this, if the jitter of our signal under study and the reference are uncorrelated, which should be the case, then

$$\left\langle J_{\text{signal}}^2 \right\rangle = \left\langle \mathcal{J}_{\text{signal}}^2 \right\rangle + \left\langle \mathcal{J}_{\text{reference}}^2 \right\rangle , \tag{15}$$

i.e., the measured mean-square jitter is larger than its actual mean-square jitter by the reference signal's mean-square jitter.

Instead of using a physical reference clock that follows the clock under study, the wander in the timings t_k can be removed using numerical methods. The idea is that a "software clock" with timings T_k is created in such a way that it follows the wander in t_k. In this way, the difference $t_k - T_k$ is the jitter (fast frequency changes) in our clock. Whether this approach is viable depends on our ability to measure the timings t_k.

For period and cycle-to-cycle jitter, while we do not need a reference signal, we still must measure the timings of the transitions. These timings won't be known exactly because of the jitter in the clock used to assign these times and timing errors due to voltage noise in the signal. Because of these errors, the measured jitter will be larger than the actual jitter of the signal. As always, whether this is acceptable depends on the requirements of the measurement.

4. Analyzing jitter

At this point, we have three types of jitter, each described by a sequence of values (all having the units of time). Normally, we would like to summarize this sequence by a single number characterizing its distribution of values.

4.1. Quantifying jitter

Usually the first step in doing this is to produce a histogram of jitter values. If this distribution is well-described by a Gaussian distribution (as is normally the case for random sources of jitter), then we can describe this distribution by its width, e.g., the root-mean-square jitter σ.

However, in general a worst-case characterization of the jitter is required, i.e. the peak-to-peak width of the distribution. While for Gaussian jitter this peak-to-peak width is unbounded in principle, we can take a multiple of the root-mean-square jitter to be sufficiently large that excursions beyond this are sufficiently rare to be allowable. A common choice is to take the peak-to-peak jitter to be 14σ.

For systems with bounded jitter, the worst-case characterization is straightforward. We simply take the peak-to-peak spread in the values.

For distributions that are neither Gaussian nor bounded (e.g., those having both bounded sources and Gaussian sources), then a modified version of the Gaussian procedure above is required. Here we look at both tails of the jitter distribution and go out far enough (e.g., using a “tail-fit” method) so that excursions beyond this are sufficiently rare to be allowable.

4.2. Random and non-random sources

When analyzing the jitter present in a signal, it is useful to divide the sources into two types. That due to random sources (e.g., noise) and that due to non-random (deterministic or systematic) sources.

Random jitter, also known as unbounded jitter or Gaussian jitter, is characterized by having a Gaussian distribution. The source for this type of jitter is usually the random noise within the system, and so eliminating these sources of jitter can be difficult.

Non-random jitter is usually characterized by a bounded non-Gaussian distribution. The source for this type of jitter is usually identifiable by analyzing the jitter spectrum. For example, the frequency of an oscillator might be modulated due to cross-talk from an adjacent signal trace or by ripples in the voltage supplying the oscillator. Since these sources are identifiable, steps can be taken to eliminate them.

4.3. Some units for jitter

As defined here, all three types of jitter naturally have units of time. For example, for systems with frequencies in the megahertz or low gigahertz range (where the period roughly a nanosecond to a microsecond), the jitter is usually expressed in picoseconds.

However, sometimes it is useful to express the jitter as its fraction of a nominal period. For example, a jitter of 2 ps for a 100 MHz (10 ns) signal would be 0.02%. This is also commonly written as 0.0002 UI, where UI stands for unit-interval.

Another convention is to express the jitter as an angular measure. As has been discussed, this makes sense for timing jitter as this can be related to the phase variations.

5. Jitter in quartz crystal oscillators

For well-designed quartz crystal oscillators with output frequencies at the frequency of oscillation of the crystal, the jitter should arise solely from random noise sources. Further, the root-mean-square period and cycle-to-cycle jitter are typically on the order of a few picoseconds (if not less). The same holds true for oscillators whose output is the binary divided output[1] of a quartz crystal oscillator.

However, the jitter can be much larger in many quartz-crystal based programmable oscillators. Such oscillators use frequency synthesis techniques (e.g., phase-locked loops) that have period and cycle-to-cycle jitter on the order of 10 ps to 100 ps (rms).

Timing jitter (phase jitter) is often quoted over a given frequency band (e.g., 12 kHz to 20 MHz for oscillators above 100 MHz) and calculated from the measured phase noise using equations (3) and (4). Because of this, timing jitter is commonly quite a bit lower than period and cycle-to-cycle jitter, e.g., less than 1 ps.

6. System induced jitter

Be aware that when installed into a complex system the jitter seen in the output signal of an oscillator can be higher than its intrinsic jitter. For example, ripples in the power supply voltage can modulate the oscillator’s frequency. If jitter performance is critical, take into account the effects of the other system components on the oscillator and take steps to minimize these effects, e.g., shielding the oscillator from electromagnetic interference and placing a simple RC-filter in its power supply line.

7. Jitter relationships

As a final thought, we use the definitions herein to explain why sometimes the measured root-mean-square cycle-to-cycle jitter is very nearly equal to the $\sqrt{3}$ times the measured root-mean-square period jitter. In particular, we show that this is the case when the jitter timings $\mathcal{J}_k$ are uncorrelated with one another.

[1] By binary divided output we mean that the output frequency is 2^{-n} times the internal oscillator frequency, for some integer $n > 0$.

Denote the average value a series x_k by $\langle x_k \rangle$. Strictly speaking, we should instead write something like $\langle x \rangle$ since the average does not depend on k, but as we'll see, it is useful for keeping track of the index we are averaging over.

If our dataset is infinite, we can exclude a finite number of members without changing the average value. As a consequence, $\langle x_{k+n} \rangle = \langle x_k \rangle$ for $n \geq 0$, since offsetting the index has the property of excluding only the first n members from the average and hence has no effect on the average. In the case of a finite dataset, we can make a similar argument that excluding a small number of members makes little difference to the average. For simplicity, we argue in the case of an infinite dataset.

Without loss in generality, suppose that our oscillator is tracking the ideal clock in the long term in the sense that

$$\langle \mathcal{J}_k \rangle = 0 . \tag{16}$$

If this is not the case, we can remove this requirement by arguing in terms of $\mathcal{J}'_k = \mathcal{J}_k - \langle \mathcal{J}_k \rangle$ instead of $\mathcal{J}_k$. (Note that $\langle \mathcal{J}'_k \rangle = 0$.)

Now suppose that the jitter timings $\mathcal{J}_k$ are uncorrelated (independent) in the sense that the average of this sequence multiplied with an offset of itself, i.e., $\langle \mathcal{J}_{k+n}\mathcal{J}_k \rangle$ satisfies

$$\langle \mathcal{J}_{k+n}\mathcal{J}_k \rangle = \langle \mathcal{J}_{k+n} \rangle \langle \mathcal{J}_k \rangle \text{, for } n > 0 , \tag{17}$$

where the average is over k. Then using equation (16), it follows that

$$\langle \mathcal{J}_{k+n}\mathcal{J}_k \rangle = 0 \text{, for } n > 0 . \tag{18}$$

With this, we have

$$\begin{aligned} \langle j_k^2 \rangle &= \langle (\mathcal{J}_{k+1} - \mathcal{J}_k)^2 \rangle \\ &= \langle \mathcal{J}_{k+1}^2 - 2\mathcal{J}_{k+1}\mathcal{J}_k + \mathcal{J}_k^2 \rangle \\ &= \langle \mathcal{J}_{k+1}^2 \rangle + \langle \mathcal{J}_k^2 \rangle \\ &= 2\langle \mathcal{J}_k^2 \rangle. \end{aligned} \tag{19}$$

In the third step, we used equation (18) and in the fourth we used the fact that offsetting the index has the property of excluding only the first member from the average and hence has no effect on the average.

Likewise

$$\begin{aligned} \langle c_k^2 \rangle &= \langle (j_{k+1} - j_k)^2 \rangle \\ &= \langle (\mathcal{J}_{k+2} - 2\mathcal{J}_{k+1} + \mathcal{J}_k)^2 \rangle \\ &= \langle \mathcal{J}_{k+2}^2 \rangle + 4\langle \mathcal{J}_{k+1}^2 \rangle + \langle \mathcal{J}_k^2 \rangle \\ &= 6\langle \mathcal{J}_k^2 \rangle. \end{aligned} \tag{20}$$

In the third step, we again used equation (18), and in the fourth we again used the fact that offsetting the index does not change the average.

Combining equations (19) and (20), we have

$$\langle c_k^2 \rangle = 3\langle j_k^2 \rangle . \tag{21}$$

So, under these circumstances, the root-mean-square cycle-to-cycle jitter is equal to $\sqrt{3}$ times the root-mean-square period jitter, i.e.,

$$c_{\text{rms}} = \sqrt{3}\, j_{\text{rms}} . \tag{22}$$

Lastly, for case considered, the measured root-mean-square period jitter allows us to calculate the root-mean-square timing jitter using

$$\mathcal{J}_{\text{rms}} = \tfrac{1}{\sqrt{2}}\, j_{\text{rms}} , \tag{23}$$

which follows from equation (19).

References

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[5] *Measurement of Oscillator Jitter*, Statek Corporation Technical Note 36, available at www.statek.com.