What is frequency at load capacitance?

1. Introduction

When ordering crystals for oscillators that are to operate at a frequency f, e.g. 32.768 kHz or 20 MHz, it is usually not sufficient to specify the frequency of operation alone. While the crystals will oscillate at a frequency near their series resonant frequency, the actual frequency of oscillation is usually slightly different from this frequency (being slightly higher in "parallel resonant circuits").[1]

So, suppose you have a crystal oscillator circuit and you want to purchase crystals such that when placed in this circuit the oscillation frequency is f. What do you need to tell the crystal manufacturer to accomplish this? Do you need to send a schematic of the oscillator design with all the associated details of its design, e.g. choice of capacitors, resistors, active elements, and strays associated with the layout? Fortunately, the answer is no. In addition to the frequency f, all that is needed is a single number, the load capacitance C_L.

2. What is C_L?

Suppose your crystal oscillator operates at the desired frequency f. At that frequency, the crystal has complex impedance Z, and for the purposes of frequency of operation, this is the only property of the crystal that matters. Therefore, to make your oscillator operate at the frequency f, you need crystals that have impedance Z at the frequency f. So, at worst, all you need to specify is a single complex number $Z = R+jX$. In fact, it is even simpler than this.

While in principal one should specify the crystal resistance R at the frequency f, usually the crystal-to-crystal variation in R and the oscillator's sensitivity to this variation are sufficiently low that a specification of R is not necessary. This is not to say that the crystal resistance has no effect; it does. We shall discuss this further in Section 4.

So, that leaves a single value to specify: The crystal reactance X at f. So, one could specify a crystal having a reactance of 400 Ω at 20 MHz. Instead, however, this is normally done by specifying a capacitance C_L and equating

$$X = \frac{1}{\omega C_L}, \qquad (1)$$

where we have set $\omega = 2\pi f$. Physically, at this frequency, the impedance of the series combination of the crystal and a capacitance C_L has zero phase (equivalently, has zero reactance or is purely resistive). See Figure 1. To see this, consider

$$\begin{aligned}(\text{Total reactance}) &= X + X_{C_L} \\ &= \left(\frac{1}{\omega C_L}\right) + \left(-\frac{1}{\omega C_L}\right) \\ &= 0,\end{aligned} \qquad (2)$$

where the second step follows by Equation (1) and the fact that the reactance of a capacitance C is $-1/(\omega C)$.

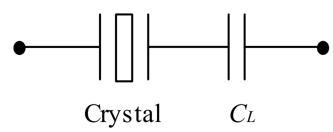

Figure 1—This series combination has zero-phase impedance at a frequency where the crystal has load capacitance C_L

So, the task of assuring proper oscillation frequency is the task of providing components (crystals in this case) that, at the specified frequency, have the required reactance, which is stated in terms of a capacitance C_L by Equation (1).[2] For example, instead of specifying crystals having a reactance of 400 Ω at 20 MHz, we specify crystals having a load capacitance of 20 pF at 20 MHz, or more normally, we specify that the crystal frequency be 20 MHz at a load capacitance of 20 pF.

In "parallel resonant circuits," C_L is positive, typically being between 5 pF and 40 pF. In this case the crystal operates in that narrow frequency band between the crystal's series and parallel resonant frequencies (F_s and F_p, respectively).

[1] When ordering crystals for series resonant operation, instead of specifying a value for C_L, be sure to state that the frequency f refers to the series-resonant frequency, F_s.

[2] This is not to say that all aspects of frequency determination are tied to this single number. For example, other aspects of the crystal and oscillator determine whether the correct mode of oscillation is selected and the system's frequency stability (short and long term).

While a truly "series resonant circuit" does not have a load capacitance associated with it [or perhaps an infinite value by Equation (1)], most "series resonant circuits" actually operate slightly off of the series resonant frequency and therefore do have a finite load capacitance (that can be positive or negative). However, if this offset is small and specifying a load capacitance is not desired, it can either be ignored or handled by a slight offset in the specified frequency *f*.

As we shall see in Section 4, both the oscillator and the crystal determine C_L. However, the crystal's role is rather weak in that in the limit of zero resistance, the crystal plays no role at all in determining C_L. In this limiting case, it makes sense to refer to C_L as the *oscillator load capacitance* as it is determined entirely by the oscillator. However, when it comes time to order crystals, one specifies crystals having frequency *f* at a load capacitance C_L, i.e. it is a condition on the crystal's frequency. Because of this, it would be reasonable to refer to C_L as the *crystal load capacitance*. For the sake of argument, we simply avoid the issue and use the term *load capacitance*.

3. Defining F_L at C_L

We now take Equation (1) as our defining relation for what we mean by a crystal having a given frequency at a given load capacitance.

Definition: A crystal has frequency F_L at a load capacitance C_L when the reactance *X* of the crystal at frequency F_L is given by Equation (1), where now $\omega = 2\pi F_L$.

Recall that, around a given mode, the reactance of a crystal increases from negative values, through zero at series resonance, to large positive values near parallel resonance where it rapidly decreases to large negative values, and then again it increases towards zero. (See Reference [1].) By excluding a region around parallel resonance, we have a single frequency for each value of reactance. In this way, we can associate a frequency F_L given a value of C_L. So, positive values of C_L correspond to a frequency between series and parallel resonance. Large negative values of C_L, correspond to a frequency below series resonance while smaller negative values correspond to frequencies above parallel resonance. (See Equation (3) below.)

3.1. The crystal frequency equation

So, how much does the frequency of oscillation depend on the load capacitance C_L? We can answer this question by determining how the crystal frequency F_L depends on the crystal load capacitance C_L. One can show that to a very good approximation that

$$F_L \approx F_s\left(1+\frac{C_1}{2\left(C_0+C_L\right)}\right), \tag{3}$$

where C_1 and C_0 are the motional and static capacitances of the crystal, respectively. (See Reference [1] for a derivation and discussion of this relation.) For the purposes of this note, we shall refer to Equation (3) as the *crystal frequency equation*. This shows the dependence of a crystal oscillator's operational frequency on its load capacitance and its dependence on the crystal itself. In particular, the fractional frequency change when changing the load capacitance from C_{L1} to C_{L2} is given to good approximation by

$$\frac{F_{L2}-F_{L1}}{F} \approx \frac{C_1}{2}\left(\frac{1}{C_0+C_{L2}}-\frac{1}{C_0+C_{L1}}\right). \tag{4}$$

3.2. Trim sensitivity

Equation (3) gives the dependence of operating frequency F_L on the load capacitance C_L. The negative fractional rate of change of the frequency with C_L is known as the trim sensitivity, *TS*. Using Equation (3), this is approximately

$$\begin{aligned} TS &= -\frac{1}{F_L}\frac{dF_L}{dC_L} \\ &\approx \frac{1}{2}\frac{C_1}{\left(C_0+C_L\right)^2}. \end{aligned} \tag{5}$$

From this we see that the crystal is more sensitive to given change in C_L at lower values of C_L.

4. But what determines C_L?

Consider the simple Pierce oscillator consisting of a crystal, an amplifier, and gate and drain capacitors as shown in Figure 2.

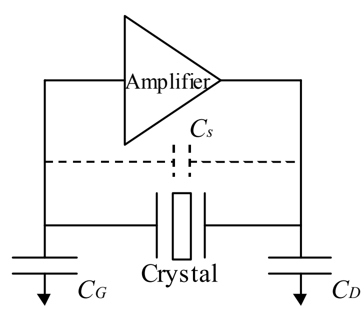

Figure 2: Simple Pierce oscillator

There are at least three stray capacitances that must be considered in trying to calculate the load capacitance of the Pierce oscillator circuit.

1. An added capacitance from the input of the amplifier to ground. Sources for this could be the amplifier itself and trace capacitance to ground. As this capacitance is in parallel with C_G, we can simply absorb this into our definition of C_G. (That is C_G is the capacitance of the capacitor to ground plus any additional capacitance to ground on this side of the amplifier.)
2. An added capacitance from the output of the amplifier to ground. Sources for this could be the amplifier itself and trace capacitance to ground. As this capacitance is in parallel with C_D, we can simply absorb this into our definition of C_D. (That is C_D is the capacitance of the capacitor to ground plus any additional capacitance to ground on this side of the amplifier.)
3. A stray capacitance C_s shunting the crystal as shown in Figure 2.

Redefining C_G and C_D as discussed above, it then follows [2] that one of the conditions for oscillation is

$$0 = X' + X_G + \left(1 + \frac{R'}{R_o}\right) X_D , \qquad (6)$$

where

$$Z' = R' + jX' \qquad (7)$$

is the impedance of the parallel combination of the crystal and the capacitance C_s and R_o is the output resistance of the amplifier.

It can be shown that the crystal resistance R as a function of load capacitance C_L is given approximately by (provided C_L is not too small)

$$R \approx R_1 \left(1 + \frac{C_0}{C_L}\right)^2 , \qquad (8)$$

where R_1 is the motional resistance of the crystal [1]. It then follows that (provided $C_L - C_s$ is not too small)

$$R' \approx R_1 \left(1 + \frac{C_0 + C_s}{C_L - C_s}\right)^2 \qquad (9)$$

and

$$X' \approx \frac{1}{\omega (C_L - C_s)} . \qquad (10)$$

With these results, Equation (6) gives the following equation for C_L

$$\frac{1}{C_L - C_s} = \frac{1}{C_G} + \frac{(1 + R'/R_o)}{C_D} , \qquad (11)$$

where R' is approximated by Equation (9). Note that the equation for C_L is actually a bit more complicated than it might seem at first as R' depends upon on C_L.

It can be seen that C_L decreases as R_1 increases, and so by Equation (3), the frequency of operation increases with crystal resistance. So, the load capacitance does have a dependence on the crystal itself. But as we have mentioned previously, the variation in crystal resistance and resulting sensitivity to this variation is usually sufficiently low that the dependence can be ignored. (In this case, a nominal value for crystal resistance is used in calculating C_L.)

However, sometimes the resistance effect cannot be ignored. Two crystals tuned so that both have exactly the same frequency at a given load capacitance C_L can oscillate at different frequencies in the same oscillator if their resistances differ. This slight difference leads to an increase in the observed system frequency variation above that due to crystal frequency calibration errors and the board-to-board component variation.

Note that in the case of zero crystal resistance (or at least negligible compared to the output resistance R_o of the amplifier), Equation (11) gives

$$C_L \approx \frac{C_G C_D}{C_G + C_D} + C_s , \quad (R_1 << R_o) . \qquad (12)$$

So, in this case, the load capacitance is the stray capacitance shunting the crystal plus the series capacitance of the two capacitances on each side of the crystal to ground.

5. Measuring C_L

While in principal one could calculate C_L from the circuit design, an easier method is simply to measure C_L. This is also more reliable since it does not rely on the oscillator circuit model, takes into account the strays associated the layout (which can be difficult to estimate), and it takes into account the effect of crystal resistance. Here are two methods for measuring C_L.

5.1 Method 1

This method requires an impedance analyzer, but does not require knowledge of the crystal parameters and is independent of the crystal model.

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1. Get a crystal that is similar to those that will be ordered, i.e. having similar frequency and resistance.
2. Place this crystal in the oscillator and measure the frequency of operation F_L. In placing the crystal into the circuit, be careful not to damage it or do anything to cause undue frequency shifts. (If soldered in place, allow it to cool down to room temperature.) A good technique that avoids soldering is simply to press the crystal onto the board's solder pads using, for example, the eraser end of a pencil and observe the oscillation frequency. Just be careful that the crystal makes full contact with the board. The system can still oscillate at a somewhat higher frequency without the crystal making full contact with the board.
3. Using an impedance analyzer, measure the reactance X of the crystal at the frequency F_L determined in Step 2.
4. Calculate C_L using Equation (1) and the measured values for F_L ($\omega = 2\pi F_L$) and X at F_L.

5.2 Method 2

This method is dependent upon the four-parameter crystal model and requires knowledge of these parameters (through your own measurement or as provided by the crystal manufacturer).

1. Get a crystal that is similar to those that will be ordered, i.e. having similar frequency and resistance.
2. Characterize this crystal. In particular measure its series frequency F_s, motional capacitance C_1, and static capacitance C_0.
3. Place this crystal in the oscillator and measure the frequency of operation F_L (as in Method 1, Step 2.)
4. Calculate C_L using Equation (3) and the measured values for F_L, F_s, C_1, and C_0.

It is recommended that either procedure be followed with at least 3 crystals. When done properly, this technique often gives values for C_L that are consistent to about 0.1 pF. Further confidence in the final results can be found by repeating the procedure for a number of boards to estimate the board-to-board variation of C_L.

Note that in the above, F_L does not have to be precisely the desired oscillation frequency f. That is, the calculated value for C_L is not a strong function of the oscillation frequency since normally only the crystal is strongly frequency dependent. If, for some reason, the oscillator does have strong frequency dependent elements, then using this procedure would be quite difficult.

6. Do I really need to specify a value for C_L?

There are at least three cases where a specification of C_L is not necessary:

1. You intend to operate the crystals at their series-resonant frequency.
2. You can tolerate large errors in frequency (on the order of 0.1% or more).
3. The load capacitance of your circuit is sufficiently near the standard value (see crystal data sheet) that the frequency difference is tolerable. This difference can be calculated with Equation (4).

If your application does not meet one of the three conditions above, you should strongly consider estimating the load capacitance of your oscillator and use this value in specifying your crystals.

7. References

1. Statek Technical Note 32.
2. Statek Technical Note 30.

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